Labyrinth Patterns in Confined Granular-Fluid Systems

B. Sandnes,* H. A. Knudsen, K. J. Måløy, and E. G. Flekkøy

Department of Physics, University of Oslo, P.O. Box 1048 Blindern, NO-0316 Oslo, Norway
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A random, labyrinthine pattern emerges during slow drainage of a granular-fluid system in two-dimensional confinement. Compacted grains are pushed ahead of the fluid-air interface, which becomes unstable due to a competition between capillary forces and the frictional stress mobilized by grain-grain contact networks. We reproduce the pattern formation process in numerical simulations and present an analytical treatment that predicts the characteristic length scale of the labyrinth structure. The pattern length scale decreases with increasing volume fraction of grains in the system and increases with the system thickness.

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Pattern and form emerge spontaneously in a diverse range of systems representing virtually every discipline of science [1]. Common denominators are that these systems are driven out of equilibrium, and that the pattern formation is a consequence of instabilities governed by competing forces [2]. Viscous fingering, for example, develops as a compromise between the Saffman-Taylor instability that induces ever more branching, and the surface tension of the fluid that puts an energetic penalty on the creation of surface area [3,4]. The variation in pattern types and length scales is striking, yet one also finds that seemingly unrelated processes may produce patterns with similar characteristics [5].

Here we demonstrate and characterize a pattern formation process where labyrinthine structures emerge during slow drainage of a fluid-grain mixture confined in a Hele-Shaw cell. The final cluster of compacted granular material is simply-connected, random, and has a characteristic wavelength. Experiments and simulations illustrated in Fig. 1 reveal that the interface instability develops as a compromise between capillary forces and granular friction, and we present an analytic prediction of the observed characteristic length scales in the patterns. Similar labyrinthine patterns have been shown to form in two-phase systems such as magnetic and dielectric fluids under external fields [6], and in some reaction-diffusion processes [7,8].

Polydisperse glass beads of diameter 50–100 μm are mixed into a 50% by volume glycerol-water solution and injected into the gap between the parallel, horizontal glass plates of a Hele-Shaw cell through a central hole in the top plate [3,9]. After injection, the fluid-grain mixture fills a circular area (~35 cm diameter) in the gap between the two plates (plate separations ranging from 0.3 to 1.0 mm). Each plate has the dimensions 50 × 50 × 1 cm. The glycerol is added in order to increase the viscosity of the fluid such that the grains remain in suspension during loading. Shortly after the cell is filled, the grains sediment out and fall to rest on the lower surface. We now start a slow drainage of the fluid in the cell through the central outlet using a syringe pump set to a withdrawal rate of typically 0.01 ml/min; thus, each experiment takes close to 3 days to complete. Due to the low flow-rate, the viscous forces that govern viscous fingering processes, are negligible.

As the fluid is gradually drained from the cell, the fluid-air interface at the perimeter of the circular disc starts to recede. The capillary forces between the wetting fluid and the grains gradually compile a growing layer of close-packed grains ahead of the interface as it moves. After a transient period where a layer of grains is formed all along the circular perimeter, the interface becomes unstable and the fluid-grain disc is slowly invaded by fingers of air. Figure 1(a) shows consecutive images of this process, where one can observe the gradual advance and splitting of fingers resulting in the final labyrinthine pattern. Unlike pinning-dominated drying processes [10,11], the finger-splitting is due to a jamming of the interface on a length scale larger than the individual grains. The grains, initially uniformly distributed, have been moved and reorganized by the invading fingers into a branching structure of close-packed particles. Each branch is assembled from the compact layer of grains pushed ahead by two adjoining fingers. While the initial circular perimeter is approximately 1 m long, the interface has stretched and deformed such that the circumference of the final cluster in Fig. 1(a), v measures 13.3 m.

The final labyrinth pattern is characterized by a wavelength that is uniform throughout the area occupied by the structure. The pattern forming process is a result of local mass transport in a direction normal to the advancing interface. The shape of the initially circular interface is continuously changing, but because the interface is stabilized and kept separate from other fingers due to frictional jamming, no pinch-off of the interface occurs during the experiment, and the interface remains a deformed circle. Consequently both the fingers and grain cluster are topologically simply connected.

The visual randomness seen in the final pattern arises from the symmetry-breaking associated with fingers going left or right. Disorder is always present in the experiments...
in form of, e.g., small inhomogeneities in mass distribution, and the dynamics of the pattern formation is governed by this, in addition to the history of the moving interface. However, simulations with no initial disorder (other than the floating point round-off errors intrinsic to the calculations) show that randomness in the pattern inevitably develops.

Changing the initial volume fraction of grains \( \varphi \) in the experiment influences the process, where the most notable effect is that the characteristic wavelength in the pattern becomes smaller as the volume fraction of grains increases [Fig. 2(a)]. The volume fraction \( \varphi \) is normalized such that \( \varphi = 1 \) for the close-packed grains. Using image analysis we identify the simply connected grain cluster in each picture and measure the characteristic wavelength as \( \lambda = 2A_{\text{disc}}/O \), where \( O \) is the circumference of the cluster and \( A_{\text{disc}} \) is the area of the initial circular disc of fluid. In the experiments \( \lambda \) ranges from 9.6 to 40.1 mm. The measurements also show a slight thickening of the grain branches with increasing volume fraction.

As in most other pattern forming systems, the instability develops as a compromise between competing forces. In the case of the granular labyrinth, the two main players are surface tension and friction. Narrow invading air fingers gather less grains and therefore encounter less friction, but the higher curvature associated with the finger tip interface gives a higher capillary force. Broader fingers, on the other hand, will encounter a smaller capillary resistance but will gather more grains along the interface, leading to increased friction. The width of the accumulated layer of grains, \( L \), is in general dependent on the initial volume fraction of grains \( \varphi \), the finger width and the history of the advancing finger. In steady state the finger gathers as much particles as it leaves behind, and the width of the layer of grains is given by \( L = \varphi \lambda/2 \).

Surface tension produces a pressure difference across a curved fluid interface. This difference, the Laplace pressure, is given by \( \Delta P_{\text{cap}} = \gamma(1/r_1 + 1/r_2) \), where \( \gamma \) is surface tension and \( r_1 \) and \( r_2 \) are the principal radii of curvature of the fluid meniscus. In the Hele-Shaw cell, the out-of-plane curvature is fixed by the parallel glass plates such that \( 1/r = 2 \cos \theta / \Delta z \), where \( \theta \) is the fluid/glass wetting angle and \( \Delta z \) is the plate separation, while the in-plane curvature may vary freely. Figure 3(a) illustrates a top view of a finger of air with a tip of curvature \( 1/R \).
The effective friction coefficient in the confined granular system is inherently difficult to estimate accurately. We have used the value $\mu = 0.47$, which is within the realistic range, and fits the experimental results at high $\varphi$. At low $\varphi$ moved length is small compared to the distance between neighboring points. The simulation is carried out with a small amount of disorder in the background mass density (distributed on a square grid). As the interface advances, the width of the compact layer is recalculated to account for the accumulated mass based on the invaded area and the background volume density of grains. As the simulation progresses, the interface stretches. When the distance between neighboring points exceeds a set limit, a new point is inserted halfway between the two original points such that the curvature of the line segment is preserved. Where two fingers meet, the respective points along the interface are immobilized as this represents the formation of a close-packed branch, and the simulation runs until the entire area defined by the initially circular disc has been compacted in this manner.

The visual similarity between simulated and experimental labyrinth patterns is evident (Fig. 1). One notable difference is the number of invading fingers across the initial circular interface (one in the simulations, several in the experiments). This can be attributed to a difference in disorder along the initial circular perimeter, which is much larger in the experiments compared to simulations. Figure 2(b) shows final labyrinth patterns generated in a series of simulations which reproduce the decreasing pattern length scale with volume fraction that we observe in the experiments. Figure 4 shows the measured characteristic wavelengths in the patterns generated in the experiments and simulations as a function of the volume fraction of grains in the systems.

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the simulations produce somewhat higher $\lambda$, possibly due to the friction law providing a less accurate description when $L$ becomes comparable to the plate spacing. The most convincing result is that the overall $\varphi$ dependencies of the simulations and experiments are very close, indicating that the rules implemented in the simulations capture the essence of the physical mechanisms. The friction law [Eq. (2)] is governed by the redistribution of stress along contact networks within the granular packing. This gives the exponentially increasing friction with $L$, which is important for reproducing the experimental results. When instead a linear friction law is implemented in the simulations, the results do not come close to agreement.

The inset in Fig. 4 shows wavelengths measured for different plate separations at constant $\varphi$. The wavelength increases with increasing system thickness, and again the results from experiments, simulations and theory are in agreement. The experiments at 0.8 and 1.0 mm plate spacing are less well defined as the hydrostatic pressure becomes significant, and the meniscus is unable to pull along all the particles, leaving behind beads on the lower plate.

To obtain a theoretical prediction of the characteristic wavelength, we assume that the moving finger tip has a constant curvature, $R$, in which case mass conservation implies $L_{up} = R\varphi/(1-\varphi)$. Using this relationship and combining Eqs. (1) and (2) give a pressure as a function of $R$ with a global minimum (surface tension dominates at high curvature, friction at low). We further assume that the fingers will form at a width that minimizes the pressure difference across the interface, i.e., $\partial \Delta P/\partial R = 0$. Inverting this equation gives the critical fingertip curvature

$$R_c = \frac{\Delta z (1 - \varphi)}{\kappa \mu \varphi} W \left( \frac{\gamma \mu \kappa^2 \varphi}{\rho g (1 - \varphi)(1 + \kappa \mu)(\Delta z)^2} \right)^{1/2}, \tag{3}$$

where $W$ is Lambert’s $W$ function. Now, we assume that the finger boundaries are straight and parallel some distance behind the curved tip. The pressure difference is the same all along the interface, such that $\Delta P(R_c) = \Delta P(L)$, where $L$ is the width of the compact layer of grains along the straight interface segments. From this we find the characteristic wavelength as $\lambda = 2L/\varphi$. The result is plotted as a function of the volume fraction in Fig. 4, and match closely the corresponding wavelength seen in the experiments and the simulations.

In conclusion, we have introduced a system that forms labyrinth patterns, and that is well understood in terms of only two competing forces: friction and the capillary pressure force. The simulations and the analytic theory capture the characteristic length scale in the experiments as a function of initial volume fraction of granular material and system thickness. The simulations also reproduce the evolution and qualitative shape of the labyrinths. The dynamics of the labyrinth formation is governed by a self-organized sequential selection of lowest interface yield pressure along the interface. The similarity of the present patterns to those of patterns formed in highly different systems, such as ferrofluids and reaction-diffusion systems, raises the question of a deeper connection.

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*bjornar.sandnes@fys.uio.no